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ASYMPTOTIC ANALYSIS OF THE PLANE CONTACT PROBLEM OF ELASTICITY THEORY FOR A TWO-LAYER FOUNDATION

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An asymptotic analysis is presented of the plane contact problem of elasticity theory for a two-layer foundation that permits selection of some model of the upper relative to a thin layer (coating), depending on the relationship between the physicomechanical and geometric values of the coating and the support (elastic half plane).

1. Let us consider an elastic half plane ($y \leqslant 0$) with Poisson ratio ν_2 and shear modulus G_2. We assume that there is a relatively thin layer $0 \leqslant y \leqslant h(\nu_1, G_1)$ on and rigidly connected to the half-plane surface.* Let a rigid stamp, for which the shape of the foundation is described by a function $f(x)$ even in x be impressed without friction by a force P on the upper boundary of such a composite medium. The boundary conditions of the problem posed are written in the form (the superscript 1 refers to the layer, and the superscript 2 to the half plane)

$$\begin{gathered} y=h\colon\ v^{(1)}=v_+(x)=-\delta+f(x),\quad \sigma_y^{(1)}=-\sigma_+(x)\quad (|x|\leqslant a),\\ \sigma_y^{(1)}=0,\ (|x|>a),\quad \tau_{xy}^{(1)}=\tau_+(x)=0\quad (|x|<\infty);\\ y=0\colon\ \sigma_y^{(1)}=\sigma_y^{(2)},\quad \tau_{xy}^{(1)}=\tau_{xy}^{(2)},\ v^{(1)}=v_-(x)=v^{(2)},\ u^{(1)}=u_-(x)=u^{(2)}. \end{gathered} \tag{1.1}$$

The stresses and strains vanish at infinity. Here δ is the rigid displacement of the stamp under the action of the force P applied thereto, $\sigma_\pm(x)$, $\tau_\pm(x)$ are the normal and tangential forces at the upper (plus sign) and lower (minus sign) faces of the layer, respectively, and $v_\pm$, $u_\pm$ are the vertical and horizontal displacements of the faces of the layer.

The formulated problem is reduced by integral transform methods [1] to the determination of the contact pressures $\sigma_+(x)$ from a convolution type integral equation of the first kind in a finite interval [2]

$$\int_{-a}^{a}\sigma_+(\xi)\,d\xi\int_{-\infty+ic}^{\infty+ic}\frac{L(\alpha)}{|\alpha|}\exp\left[-i\frac{\alpha}{h}(\xi-x)\right]d\alpha=2\pi\theta_1[\delta-f(x)]\quad (|x|\leqslant a); \tag{1.2}$$

$$L(u)=\frac{M+4|u|e^{-2|u|}-Ne^{-4|u|}}{M-(1+4u^2+NM)e^{-2|u|}+Ne^{-4|u|}}, \tag{1.3}$$

$$\mu_i=1-\nu_i^*,\ \varkappa_i=3-4\nu_i,\ \theta_i=G_i\mu_i^{-1}\ (i=1,2),\ n=\theta_1\theta_2^{-1},$$
$$M=(n\mu_1+\mu_2\varkappa_1)(n\mu_1-\mu_2)^{-1},\ N=(n\mu_1\varkappa_2-\mu_2\varkappa_1)(n\mu_1\varkappa_2+\mu_2)^{-1}.$$

Taking account of the notation

$$\begin{gathered} u=\alpha h,\ x=x'a,\ \xi=\xi'a,\ \lambda=ha^{-1},\\ \sigma_+(x)\,\theta_1^{-1}=q(x'),\ \delta=\Delta a,\ f(x)=r(x')\,a \end{gathered} \tag{1.4}$$

*We call a layer thin if the dimensionless parameter is $\lambda = ha^{-1} \ll 1$, where $2a$ is the loading section of the strip.

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we represent (1.2) in the form (primes are omitted)

$$\int_{-1}^{1} q(\xi)\, k\left(\frac{\xi - x}{\lambda}\right) d\xi = 2\pi [\Delta - r(x)] \quad (|x| \leqslant 1),$$
$$k(z) = \int_{-\infty+ic}^{\infty+ic} \frac{L(u)}{|u|} e^{-iuz} du \quad \left(z = \frac{\xi - x}{\lambda}\right). \tag{1.5}$$

Here the function L(u) is continuous, real, and even on the real axis, and also

$$L(t) = 1 + O(e^{-2|t|}) \ (t \to \infty), \ L(t) = n + A|t| + O(t^2) \ (t \to 0),$$
$$A = -\frac{(1-2\nu_2)^2}{2\mu_2^2}\left[n^2 + 2n\frac{\nu_1\mu_2}{\mu_1(1-2\nu_2)} - \frac{(1-2\nu_1)^2\mu_2^2}{(1-2\nu_2)^2\mu_1^2}\right], \ u = t + i\tau.$$

We will describe the physicomechanical properties of the thin layer by using refined equations of plates. To derive them we use the solution of the first boundary-value problem of elasticity theory for a strip, i.e., we find the solution of the elasticity theory equations in displacements (Lamé equations) for the following boundary conditions on its faces

$$y = h: \ \sigma_y^{(1)} = \sigma_+(x), \quad \tau_{xy}^{(1)} = \tau_+(x); \quad y = 0: \ \sigma_y^{(1)} = \sigma_-(x), \quad \tau_{xy}^{(1)} = \tau_-(x).$$

Applying the Fourier integral transform in x as above, we obtain

$$u_x'(x, y) = -\frac{i}{2\pi}\int_{-\infty}^{\infty} [c_1(\alpha)\operatorname{ch}\alpha y + c_2(\alpha)\alpha y \operatorname{sh}\alpha y + d_1(\alpha)\operatorname{sh}\alpha y + d_2(\alpha)\alpha y\operatorname{ch}\alpha y]\,\alpha e^{-i\alpha x} d\alpha; \tag{1.6}$$

$$v_x''(x, y) = -\frac{i}{2\pi}\int_{-\infty}^{\infty} \{[d_1(\alpha) - \varkappa_1 d_2(\alpha)]\operatorname{ch}\alpha y + d_2(\alpha)\alpha y\operatorname{sh}\alpha y + [c_1(\alpha) - \varkappa_1 c_2(\alpha)]\operatorname{sh}\alpha y + c_2(\alpha)\alpha y\operatorname{ch}\alpha y\}\,\alpha^2 e^{-i\alpha x} d\alpha, \tag{1.7}$$

$$c_j(\alpha) = [2iG_1\alpha D_+(\alpha)]^{-1}\{[\Sigma_+(\alpha) + \Sigma_-(\alpha)]S_j(\alpha) + i[T_+(\alpha) - T_-(\alpha)]C_j(\alpha)\},$$
$$d_j(\alpha) = [2iG_1\alpha D_-(\alpha)]^{-1}\{[\Sigma_+(\alpha) - \Sigma_-(\alpha)]S_j^*(\alpha) + i[T_+(\alpha) + T_-(\alpha)]C_j^*(\alpha)\},$$
$$D_\pm(\alpha) = \operatorname{sh}\alpha h \pm \alpha h, \quad S_1(\alpha) = (\alpha h/2)\operatorname{ch}(\alpha h/2) - (1-2\nu_1)\operatorname{sh}(\alpha h/2),$$
$$S_2(\alpha) = -\operatorname{sh}(\alpha h/2), \quad C_1(\alpha) = 2\mu_1\operatorname{ch}(\alpha h/2) - (\alpha h/2)\operatorname{sh}(\alpha h/2),$$
$$C_2(\alpha) = \operatorname{ch}(\alpha h/2), \quad S_1^*(\alpha) = (\alpha h/2)\operatorname{sh}(\alpha h/2) - (1-2\nu_1)\operatorname{ch}(\alpha h/2),$$
$$S_2^*(\alpha) = -\operatorname{ch}(\alpha h/2), \quad C_1^*(\alpha) = 2\mu_1\operatorname{sh}(\alpha h/2) - (\alpha h/2)\operatorname{ch}(\alpha h/2), \quad C_2^*(\alpha) = \operatorname{sh}(\alpha h/2).$$

Here $\Sigma_\pm(\alpha)$ and $T_\pm(\alpha)$ are, respectively, the Fourier transforms of the functions $\sigma_\pm(x)$ and $\tau_\pm(x)$.

As $\lambda \to 0$, by simplifying (1.6) and (1.7), written in Fourier transform symbols, and then returning to the originals while taking into account that $u \sim \tau h$, $v \sim \sigma h$ in contact problems (see the degenerate solutions for a layer of small thickness, say [1]), we write

$$G_1 h^2 u_\pm'' = -\nu_1\frac{h^2}{4}(\sigma_+'' + \sigma_-'') - \mu_1\frac{h}{2}\left[\tau_+' - \tau_-' - \frac{3h^2}{10}(\tau_+''' - \tau_-''')\right] \mp$$
$$\mp 3\mu_1(\sigma_+ - \sigma_-) \pm (2 - 3\nu_1)\frac{h^2}{4}(\sigma_+'' - \sigma_-'') \mp 3\mu_1\frac{h}{2}\left[\tau_+' + \tau_-' - \frac{h^2}{6}(\tau_+''' + \tau_-''')\right]; \tag{1.8}$$

$$G_1 h^3 v_\pm^{(4)} = 6\mu_1\left[\sigma_+ - \sigma_- - \frac{h^2}{3}(\sigma_+'' - \sigma_-'') + \frac{11h^4}{240}(\sigma_+^{(4)} - \sigma_-^{(4)})\right] +$$
$$+ 3\mu_1 h(\tau_+' + \tau_-') - (2 - 3\nu_1)\frac{h^3}{4}(\tau_+''' + \tau_-''') \pm \mu_1\frac{h^4}{8}(\sigma_+^{(4)} + \sigma_-^{(4)}) \pm \nu_1\frac{h^3}{4}(\tau_+''' - \tau_-'''). \tag{1.9}$$

Let us note that (1.8) and (1.9) permit taking into account both the deformation of the longitudinal tension and the transverse shear and the deformation of the longitudinal shear and the transverse compression of the elastic coating (plate). Moreover, utilization of (1.8) and (1.9) for the solution of contact problems does not result in the appearance of lumped force sections on the connecting boundaries as when using the elasticity theory equations as well as (1.10) and (1.11). As is known [3-6], this disadvantage is inherent to the differential equations of thin-walled elastic element bending, obtained on the basis of the Kirchhoff–Love, Reissner hypotheses or their modifications.

If the displacements has been averaged with respect to the thickness in (1.8) and (1.9) because of the smallness of the parameter $\lambda = h\alpha^{-1}$, then we would arrive at the following simplified equations of plate deformation

$$4G_1 h u_*'' = -2\mu_1(\tau_+ - \tau_-) - \nu_1 h(\sigma_+' + \sigma_-') + (1 - 2\nu_1)(h^2/6)(\tau_+'' - \tau_-''); \tag{1.10}$$

$$4G_1 h^3 v_*^{(4)} = 12\mu_1 h(\tau_+' + \tau_-') - \mu_1 h^3(\tau_+''' + \tau_-''') + 24\mu_1(\sigma_+ - \sigma_-) - 2(3 - 2\nu_1)h^2(\sigma_+'' - \sigma_-'') + (3 - 2\nu_1)(h^4/10)(\sigma_+^{(4)} - \sigma_-^{(4)}), \tag{1.11}$$

which, in contrast to (1.8) and (1.9), take account only of the longitudinal tension and transverse bending deformations.†

2. Let us consider the problem formulated in Sec. 1 by using different versions of the mechanical models of an elastic layer. We assume that its stress–strain state is characterized by (1.8) and (1.9). Applying the Fourier transform in the coordinate x to the Lamé equations which are used to describe the deformations of the elastic half plane, and to (1.8) and (1.9) with conditions (1.1) and the notation (1.4) taken into account, we obtain an integral equation of the form (1.5) in the unknown contact pressures under the stamp, where

$$L(u) = \left[n + \sum_{k=1}^{5} a_{k1}|u|^k\right]\left[1 + \sum_{k=1}^{4} b_{k1}|u|^k\right]^{-1},$$

$$a_{11} = (1 - 2\nu_1)(2\mu_1^2)^{-1} - n\nu_1(1 - 2\nu_2)(\mu_1\mu_2)^{-1} + n^2\varkappa_2(2\mu_2^2)^{-1}, \quad a_{21} = 32n/15,$$

$$a_{31} = (34 - 53\nu_1 + 9\nu_1^2)(60\mu_1^2)^{-1} + n(3 - 7\nu_1)(1 - 2\nu_2)(12\mu_1\mu_2)^{-1} + n^2\varkappa_2(6\mu_2^2)^{-1},$$

$$a_{41} = 47n/60, \quad a_{51} = (22 - 39\nu_1 + 13\nu_1^2)(120\mu_1^2)^{-1} + n(5 - 13\nu_1)(1 - 2\nu_2)(120\mu_1\mu_2)^{-1} + n^2\varkappa_2(30\mu_2^2)^{-1}, \quad b_{11} = 2n,$$

$$b_{21} = (17 - 2\nu_1)(15\mu_1)^{-1} + n(1 - 2\nu_2)\mu_2^{-1}, \quad b_{31} = 4n/3,$$

$$b_{41} = (27 - 44\nu_1 + 12\nu_1^2)(60\mu_1^2)^{-1} + n(1 - 2\nu_1)(1 - 2\nu_2)(6\mu_1\mu_2)^{-1} + n^2\varkappa_2(12\mu_2^2)^{-1}.$$

We now take the plate bending equation of Reissner type [3], obtained from (1.11) if the third component in its right side is neglected, as the mechanical model of the coating

$$G_1 h^3 v_*^{(4)} = 6\mu_1(\sigma_+ - \sigma_-) + 3\mu_1 h\tau_-' - \frac{3 - 2\nu_1}{2}h^2(\sigma_+'' - \sigma_-'') - \mu_1\frac{h^3}{4}\tau_-'''. \tag{2.1}$$

Moreover, we consider the horizontal displacement in the thin layer described by the equation of uniaxial tension of the plate, i.e., the differential equation (1.10) in which the component of order λ^2 has been eliminated in the right side

$$4G_1 h u_*'' = 2\mu_1\tau_- - \nu_1 h(\sigma_+' + \sigma_-'). \tag{2.2}$$

It is taken into account in (2.1) and (2.2) that $\tau_+ \equiv 0$. Setting

$$v_+ = v_- = v_*, \quad u_+ = u_- = u_* \tag{2.3}$$

into (1.1), we arrive at the integral equation (1.5) where

$$L(u) = \left[n + \sum_{k=1}^{4} a_{k2}|u|^k\right]\left[1 + \sum_{k=1}^{4} b_{k2}|u|^k\right]^{-1},$$

$$a_{12} = n^2\varkappa_2(2\mu_2^2)^{-1} - n(1 - 2\nu_2)\nu_1(2\mu_1\mu_2)^{-1}, \quad a_{22} = n(3 - 5\nu_1)(12\mu_1)^{-1},$$

$$a_{32} = n(3 - 2\nu_1)(24\mu_1\mu_2)^{-1}\left[n\varkappa_2\mu_2^{-1} - (1 - 2\nu_2)\nu_1\mu_1^{-1}\right], \quad a_{42} = -n\nu_1(8\mu_1)^{-1},$$

$$b_{12} = 2n, \quad b_{22} = (3 + \nu_1)(12\mu_1)^{-1} + n(1 - 2\nu_2)(2\mu_2)^{-1}, \quad b_{32} = n(4 - 3\nu_1)(6\mu_1)^{-1},$$

$$b_{42} = n^2\varkappa_2(12\mu_2^2)^{-1} + n(1 - 2\nu_1)(1 - 2\nu_2)(24\mu_1\mu_2)^{-1} + \nu_1(48\mu_1)^{-1}.$$

Furthermore, let the physicomechanical properties of the coating be described by the Kirchhoff–Love equations for a plate [3] obtained from (1.11) by discarding components of order λ^2 and higher in its right side

$$G_1 h^3 v_*^{(4)} = 6\mu_1(\sigma_+ - \sigma_-) + 3\mu_1 h\tau_-'$$

and by (2.2). In conformity with (1.1) and (2.3), the problem results in finding the unknown contact pressures under the stamp from the integral equation (1.5), where

$$L(u) = \left[n + \sum_{k=1}^{2} a_{k3}|u|^k\right]\left[1 + \sum_{k=1}^{4} b_{k3}|u|^k\right]^{-1},$$

†Equations (1.8)-(1.11) were obtained jointly with V. M. Aleksandrov.

$$a_{13} = n^2\varkappa_2 (2\mu_2^2)^{-1}, \quad a_{23} = -n\nu_1 (8\mu_1)^{-1}, \quad b_{13} = 2n,$$
$$b_{23} = n(1-2\nu_1)(2\mu_2)^{-1} + \nu_1 (4\mu_1)^{-1}, \quad b_{33} = n/6, \tag{2.4}$$
$$b_{43} = n^2\varkappa_2 (12\mu_2^2)^{-1} - n(1-2\nu_2)\nu_1 (24\mu_2)^{-1}.$$

Let us still examine the following simplified version of (1.10) and (1.11). Neglecting terms of order λ^2 and above therein, we obtain

$$4G_1 h u_*'' = 2\mu_1 \tau_- - \nu_1 h (\sigma_+' + \sigma_-'), \quad \sigma_+ = \sigma_- - (1/2) h \tau_-'.$$

Analogously to the previously elucidated, we arrive at an integral equation of the form (1.5) where

$$L(u) = \left[n + \sum_{k=1}^{2} a_{k4} |u|^k\right]\left[1 + \sum_{k=1}^{2} b_{k4} |u|^k\right]^{-1},$$
$$a_{14} = n(2\mu_2)^{-1}\left[n\varkappa_2\mu_2^{-1} - (1-2\nu_2)\nu_1\mu_1^{-1}\right], \quad a_{24} = n^2(1-2\nu_2)(2\mu_2)^{-1} + n\nu_1(4\mu_1)^{-1}, \tag{2.5}$$
$$b_{14} = 2n, \quad b_{24} = n(1-2\nu_2)(2\mu_2)^{-1} + \nu_1(4\mu_1)^{-1}.$$

Let the layer whose stress–strain state is described by (1.8) and (1.9) lie on a rigid foundation, i.e., set

$$y = 0: v^{(1)} = 0, \ u^{(1)} = 0 \tag{2.6}$$

in (1.1). Then the boundary-value problem (1.8), (1.9), (1.1), (2.6) is equivalent to the integral equation (1.5) whose kernel has the form

$$k(z) = 2\int_0^\infty L(u)\cos uz\,du \tag{2.7}$$

$$L(u) = \sum_{k=0}^{2} a_{k5} u^{2k}\left(\sum_{k=0}^{2} b_{k5} u^{2k}\right)^{-1},$$

$$a_{05} = \frac{1-2\nu_1}{2\mu_1^2}, \quad a_{15} = \frac{34 - 53\nu_1 + 9\nu_1^2}{60\mu_1^2}, \quad a_{25} = \frac{22 - 39\nu_1 + 13\nu_1^2}{120\mu_1^2}, \tag{2.8}$$
$$b_{05} = 1, \quad b_{15} = (17 - 2\nu_1)(15\mu_1)^{-1}, \quad b_{25} = (27 - 44\nu_1 + 12\nu_1^2)(60\mu_1^2)^{-1}$$

as is easy to show by using the method elucidated.

Taking account of the behavior of the Fourier transforms (2.8) at infinity and using the properties of the Dirac delta function [7], Eqs. (1.5) and (2.7) can be written in the form

$$n_1 q(x) + \int_{-1}^{1} q(\xi)\, l\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi[\Delta - r(x)] \quad (|x| \leqslant 1),$$
$$l(z) = \int_0^\infty L(u)\cos uz\,du, \quad L(u) = (a_{06} + a_{16}u^2)\left(\sum_{k=0}^{2} b_{k5} u^{2k}\right)^{-1}, \tag{2.9}$$
$$n_1 = \pi\lambda a_{25} b_{25}^{-1}, \quad a_{06} = a_{05} - a_{25} b_{25}^{-1}, \quad a_{16} = a_{15} - a_{25} b_{15} b_{25}^{-1}.$$

It should here be kept in mind that if $\tau_+ = 0$, $u_- = v_- = 0$, then to the accuracy of terms in $O(\lambda)$, a Voss–Winkler foundation equation is obtained from (1.8) and (1.9) that describes the deformation of transverse compression of the coating:

$$2G_1\mu_1 v_+ = (1 - 2\nu_1) h \sigma_+, \quad \sigma_+ = \sigma_-. \tag{2.10}$$

Now if the physicomechanical properties of the thin layer are characterized by (2.10) then the contact problem formulated in Sec. 1 for a two-layer foundation can be reduced to an integral equation of the mixed problem for a foundation by the Fourier integral transform method [9]

$$\lambda \frac{1-2\nu_1}{2\mu_1^2} q(x) + \frac{n}{\pi}\int_{-1}^{1} q(\xi)[-\ln|\xi - x| + D]\, d\xi = \Delta - r(x) \quad (|x| \leqslant 1), \tag{2.11}$$

where D is an arbitrary constant. It will be determined if the half plane is considered a layer of large thickness H. Then $D = \ln Ha^{-1} + a_0$, where $a_0 = -0.352$ when the layer lies on a rigid foundation without friction, and $a_0 = -0.527$ when the layer is clamped rigidly on the foundation ($\nu_2 = 0.3$) [1].

Finally, in the case when the layer (2.10) lies on a rigid foundation, we arrive at an integral equation of the form (1.5), where

$$L(u) = (1-2\nu_1)(2\mu_1^2)^{-1}|u|, \tag{2.12}$$

from which the expression for the contact pressures will be given by the formula

$$q(x) = 2\mu_1^2\lambda^{-1}(1-2\nu_1)^{-1}[\Delta - r(x)] \quad (|x| \leqslant 1).$$

Moreover, the statics condition to set up a connection between the quantities P and δ

$$P(\theta_1 a)^{-1} = \int_{-1}^{1} q(x)\,dx$$

should be added to the integral equations obtained for the contact problem about the impression of a stamp in a composite foundation (1.5), (2.9), (2.11).

Let us note that if the contact zone between the stamp and the coating a is unknown in advance (the stamp has no corners) additional conditions must be formulated to determine it.

The additional conditions for the coating being modelled by the Kirchhoff–Love plate [8] will be

$$v_*''(\pm 1) = -r''(\pm 1).$$

For the remaining models it is necessary to take account of the continuity conditions $\sigma_+(x)$ for the stress during passage through the point $x = \pm a$, $q(\pm 1) = 0$.

3. We now consider particular cases of the formulated problem for (1.5) by considering

$$n \sim \lambda^m \ (\lambda \to 0). \tag{3.1}$$

Here the stiffness of the elastic half plane is greater than the coating stiffness for $m > 0$, and conversely for $m < 0$.

Let $m = 0$. Substituting (3.1) into (1.3), we obtain to the accuracy of terms of the order of $O(\lambda^3)$ the expression (2.1) which corresponds to the case of a contact problem for a two-layer foundation when the physicomechanical properties of the coating are described by (1.8) and (1.9).

For $m = 1$ we have from (1.3) with (3.1) taken into account to the accuracy of terms of the order of $O(\lambda^2)$

$$L(u) = (1-2\nu_1)(2\mu_1^2)^{-1}|u| + n.$$

As is seen from (2.10), in this case the coating works on a Voss–Winkler type foundation.

For $m \geqslant 2$, we obtain an expression of the form (2.12) by discarding terms on the order of $O(\lambda^2)$ in (1.3), i.e., the whole two-layer packet works on a Voss–Winkler type foundation with bedding coefficient $2G_1\mu_1 h^{-1}(1-2\nu_1)^{-1}$.

We now allow $m \geqslant 6$. Substituting (3.1) into (1.3), we arrive at a contact problem for a rigidly clamped layer at the lower boundary, whose physicomechanical properties are described by (1.8) and (1.9) to the accuracy of terms of the order $O(\lambda^2)$.

Furthermore, let us consider the case when the rigidity of the coating is greater than the rigidity of the foundation.

Setting $m = -1$, according to (3.1) and (1.3) we obtain an expression of the form (2.5) to the accuracy of terms $O(\lambda)$, i.e., the coating will operate as a kind of plate. For $m \leqslant -2$, substituting (3.1) into (1.3) and keeping terms of order up to $O(\lambda^2)$, we will have

$$L(u) = n\left(1 + n\frac{\varkappa_2}{2\mu_2^3}|u|\right)\left[1 + 2n\left(|u| + \frac{1-2\nu_2}{2\mu_2}u^2\right) + n^2\frac{\varkappa_2}{12\mu_2^2}u^4\right]^{-1}. \tag{3.2}$$

The expansion (3.2) agrees with the corresponding expression (2.4) to the accuracy of terms $O(\lambda^2)$. Therefore, to a given accuracy the physicomechanical properties of a coating can be modeled by the bending equations of Kirchhoff–Love plates. Analogously neglected terms on the order of $O(\lambda^2)$ for $m = -2$, $O(\lambda^3)$ for $m = -3$, etc., we obtain the kernel of the integral equation for the contact problem about stamp interaction with an elastic half plane through a plate of Reissner type.

Let us note that when the physicomechanical properties of the coating are described by (1.8) and (1.9), the function (2.1) will agree with the expansion (1.3) to the accuracy of terms $O(\lambda^2)$ inclusive for m = −2, and to $O(\lambda^3)$ for m = −3, etc.

Finally, for m = −7, expression (1.3) to the accuracy of terms $O(\lambda^4)$ takes the form corresponding to the problem about stamp interaction with a layer whose lower face is free of forces (the presence of an appropriate layer overload outside the stamp must be assumed for the correct formulation of such a problem).

Thus, depending on the relationships between the coating and foundation stiffnesses and the order of the terms in λ retained in the Fourier transform of the kernel (1.3), functions L(u) were obtained that correspond to contact problems about stamp interaction with an elastic half plane through a plate described by (1.8) and (1.9), through a plate of Reissner type, a Kirchhoff–Love plate, a coating operating on the plate cover type, a layer of Winkler springs. Moreover, the passage to the limit case of impression of a stamp on a coating of the underlying rigid foundation is possible.

In conclusion, we note that the asymptotic analysis presented permits selection of any coating model depending on the relationship between the physicomechanical and the geometrical characteristics with respect to the thin layer and the half plane, which is more preferrable in the majority of cases than the use of theory of elasticity equations. As a result of the analysis, it is seen that the refined plate model described by (1.8) and (1.9) corresponds sufficiently well to the exact solution of the problem in the whole range of variation of the change in the parameter m.

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